Free Fall

Cycle I

1. Introduction

1.1. Objective

The experimenter will measure the acceleration due to gravity on Earth by two methods, for free-falling objects with different masses.

1.2. Equipment

1. Free fall apparatus, mounted on table
2. Grey pad (a switch)
3. Electronic timer (a black box that plugs into the wall)
4. Two metal balls of different sizes
5. Meter stick, tape measure, or other tool for measuring distance

1.3. Discussion

1.3.1 Galileo Galilei

According to an old story, Galileo dropped spheres of different masses off the leaning tower of Pisa to prove that objects do not, as reported by Aristotle, fall with accelerations proportional to their masses. There is strong evidence[1] that Galileo never did the experiment as described in the story; instead, he probably gathered his data in an equivalent, but much slower, manner using inclined planes. We will use modern timing equipment to perform the experiment as described in the traditional story.

Galileo showed that an object falling freely in a uniform gravitational field is accelerated at a constant rate. The force[2] that causes the acceleration is the result of the mutual gravitational attraction between the falling mass and the Earth. Of course, if there are other forces present, such as friction or air resistance, the motion of the falling object will not be one of constant acceleration. However, if the distance of fall is not too great and the object is sufficiently dense, the effects of air resistance are very small and may be ignored.

1.3.2 Derivation of an Equation for the Acceleration

An accelerating object feels a force described by Equation 1, Newton's second law, in which $\vec{a}$ is the object's acceleration, and m is its mass.

$$\vec{F} = m\vec{a} \tag{1}$$

This equation can be re-arranged to solve for the acceleration $\vec{a}$ in terms of the mass m and the force, $\vec{F}$:

$$\vec{a} = \frac{\vec{F}}{m} \tag{2}$$

If the force is equated with the gravitational attraction between the Earth and a falling object of mass m[3],

[1] Citation needed

[2] It is worth mentioning that Galileo died in 1642, the year Newton was born, so he did not know about forces or Newton's laws of motion.

$$\vec{F} = G\frac{mM_\oplus}{R_\oplus}\hat{r} \tag{3}$$

and this is inserted into Equation 2 to find the object's acceleration[4],

$$\vec{a} = -G\frac{M_\oplus}{R_\oplus}\hat{y} \tag{4}$$

or

$$\vec{a} = -g\hat{y} \tag{5}$$

This g is called the *acceleration due to gravity (on Earth)*[5]. It is approximately constant near the surface of the Earth, as Galileo discovered, where it has a value close to 9.81 m/s^2.

The path of an object in motion under constant acceleration is described by the kinematic equation[6]

$$\vec{x}(t) = \vec{x_0} + \vec{v_0}t + \frac{1}{2}\vec{a}t^2 \tag{6}$$

where x_0 and v_0 are the position and velocity of the object at the beginning of an experiment. If we choose to set the initial position x_0 to some height h above the ground and drop the object from a rest state in which $v_0 = 0$, Equation 6 becomes

$$y(t) = h - \frac{1}{2}gt^2 \tag{7}$$

If the object lands on the ground at $y(t) = 0$, then Equation 7 can be solved for g in terms of the time t that the object takes to fall to the ground and its initial height h

$$g = \frac{2h}{t^2} \tag{8}$$

In the two experiments described below, you will carefully measure the time t it takes a free-falling object to fall a known distance h in order to calculate the acceleration due to gravity on Earth, g.

2. Performing the Experiment

By dropping balls with different masses a known distance and measuring their fall times, one can calculate their accelerations by Equation 8. About 400 years ago, Galileo showed that the acceleration experienced by all free-falling objects is independent of their mass. In this experiment, you will do the same.

Note that the most efficient way to take your data is to make all of your time measurements *for both balls* at the same height before changing the apparatus.

2.1. Procedure

Note that if you are performing this experiment with two metal balls, it is most efficient to take all of the data for *both balls* at a given height before changing the apparatus.

1. Convince yourself that the two metal balls have different masses. (You may use a lab balance to assist you in this endeavor, if you wish.)

2. Assemble the timer with the free fall adapter so the steel ball has an unobstructed path to the grey pad.

[3] The symbol "⊕" used here means "Earth", so $M_\oplus$ and $R_\oplus$ represent the mass and the radius of the Earth, respectively.

[4] Note that in our lab, $\hat{r}$, the unit vector pointing from the object toward the center of the Earth, is the same as $-\hat{y}$, the unit vector pointing down.

[5] The same equations work just as well on another planet, or an asteroid, or perhaps on the back of a giant turtle, if the correct mass and radius are used.

[6] This is actually *three* equations: one for each of the x-, y-, and z-directions, all described by the vector $\vec{x}$. Since the full vector equation only has a y-component, it reduces to a single equation for y.

3. Push the spring-held rod in slightly, and place the ball between the two metal contacts on the drop mechanism. Turn the thumbscrew so that the ball is held in place. You can then release the ball by loosening the screw.
4. Adjust the height of the ball to about 1.25 m. **Measure the height** from the bottom of the ball when held in the ball-dropping mechanism to the top of the grey pad **and record your measurement.**
5. Set the timer modes to "Stopwatch." Press the "Start/Stop" key. The timer will beep and "*" will appear on the second line of the LCD. Dropping the ball will start the timer. When the ball hits the grey pad the timer will stop and display the elapsed time.
6. Practice dropping the ball several times to ensure that the ball hits the pad near the center before you begin the experiment.
7. Drop the ball at least five times and **record the time measurements.**
8. **Repeat steps 5–7 for the second ball**.
9. **Repeat steps 4–8 for four more heights** (for example, 0.25, 0.50, 0.75, and 1.00 meters). It is not important that the heights be exactly at the specified values, but you should measure the heights as accurately as you can. This measurement probably contributes the largest source of uncertainty to your final result.

2.2. Analysis

Perform all of the following steps for each ball.

1. For each time at each height, **compute and record the value of *g* using Equation 8**.
2. For each value of g that you calculated in the previous step, **calculate g^2** and record it.
3. Calculate the average value of g from your list of results. Recall that the average $\langle g \rangle$ (also known as the *mean*) can be calculated as the sum of a list of values g_i, divided by the number of entries N in the list (Equation 9).

$$\langle g \rangle = \frac{1}{N} \sum_{i=1}^{N} g_i \tag{9}$$

   The average value of g, $\langle g \rangle$, is half of your final result.

4. Similarly to the previous step, calculate the average value of g^2.
5. Now you will **calculate** a measure of the uncertainty in your measurements known as **the standard deviation**. In order to do this, you must first **compute the variance** (Equation 10); the standard deviation is equal to the (positive!) square root of the variance (Equation 11).

$${\sigma_g}^2 = \langle g^2 \rangle - \langle g \rangle^2 \tag{10}$$

$$\sigma_g = \sqrt{\sigma_g^2} \tag{11}$$

   Equation 10 can be read as: *the variance equals the mean of squares minus the square of the mean.*

   Calculate both the variance and standard deviation from your list of values for g and g^2. The standard deviation is the second half of your final result.

6. From the standard deviation, you can **determine the relative uncertainty** in your measurement of g. Divide the standard deviation by the average value of g to find the relative uncertainty (Equation 12).

$$\text{relative uncertainty} = \frac{\sigma_g}{\langle g \rangle} \tag{12}$$

7. **Compare your average values of *g* for each ball to the standard value of 9.81 m/s^2 by computing the percent difference.** The percent difference is given by the difference between your result and the standard value divided by the standard value and converted to a percent:

$$\text{percent difference} = 100\% \times \frac{g_{measured} - g_{standard}}{g_{standard}} \tag{13}$$

If your relative uncertainty is greater than your percent difference, then you can claim that your measured value of g agrees with the standard value, within the limits of your uncertainty. **Does your measured value of *g* agree with the standard value within your uncertainty?**

3. Questions

Answer the following questions in your report.

1. **What is your measured result for the acceleration due to gravity on Earth *g* for each ball?** Report it as

$$\langle g \rangle \text{ m/s}^2 \pm \sigma_g \text{ m/s}^2,$$

making note of the uncertainty in your measurements.

2. How well do your measurements compare with the standard value of 9.81 m/s^2 for the gravitational acceleration? **Did you successfully measure the standard value, within your uncertainties?**

3. What factors contribute to any differences between what you have found and what you expected?