FORCE TABLE

OBJECTIVE

To study vector addition by comparing the results of the experimental addition of several forces with the results of graphical constructions.

EQUIPMENT

Force table w/ pin, 3-pulleys, 3-weight hangers, masses, protractors.

INTRODUCTION

Notice that the rim of the force table is ruled in degrees and that there are clamps with pulleys that can be set at various angles around the table. Masses can be hung on cords that pass over the pulleys and connect to a small ring in the center of the table. The gravitational force on the masses (their weight) is applied to the ring by the cords passing over the pulleys. The pulleys can be repositioned around the force table and the masses changed until the system is balanced (i.e., in equilibrium). Then the angle between pulleys can be read directly from the calibrated table edge.

A pin is supplied with the table to hold the ring near the center of the table while different weights and angles are tried for balance. When the forces are in balance, the ring will be centered around the pin; a small displacement will result in the ring oscillating about but it will settle around the pin. In this state the pin can be removed and larger amplitude oscillations tried; the ring will come to rest over the position of the pin if the system is in equilibrium.

TWO VECTORS

1. Place two pulleys opposite each other at 37 and 217 degrees. Pin the ring to the center of the table. Load the two weight holders with unequal masses, to total 100 grams on one and 150 grams on the other. Be sure to include the mass of the holder. Notice that it is impossible to cause the ring to stay centered around the pin unless you use your hand to apply an additional force downward on the smaller mass.

2. Add to the 100 gram mass to equal the companion mass of 150 grams. Notice now that the ring can be set in a stable position

about the pin indicating a balance of forces. The two forces on the ring are equal in magnitude but opposite in direction.

THREE VECTORS

Symmetric Situation

3. Position three pulleys such that they are 120 degrees apart. The angles 0, 120, and 240 degrees work well for this. With the ring pinned to the center place 150 grams on each holder. This should be an equilibrium situation. When you think you have a balance, test it by removing the pin. Then lift the ring slightly and let go. It will oscillate and allow the weights to move. If you have achieved equilibrium, the ring should return to its position centered over the pin location.

Error Analysis

4. Choose one pulley and move it slightly away from its current position. There is probably a small range of angles within which you can move the pulley without sensibly disturbing the equilibrium. This range of angles represents your uncertainty in the true position of the pulley. Record this range of angles as $\Delta\theta$.

5. Return the pulley to the centered position. Now add a small amount of mass to the hanger. At some added mass, the equilibrium will be disturbed. The amount of mass added that causes this represents your uncertainty in the true magnitude of the force. Record this mass as Δm.

6. There are two ways to represent the uncertainties in each force: graphically or via an equation. Since you will be required to perform analysis in both situations, you will need to handle the uncertainties both ways.

Graphical Error Analysis

When you draw a vector, you use a length scale to determine the magnitude and a reference line to determine the angle. Due to the uncertainties, each vector you draw should have a range of lengths and a sweep of angles. Thus, the tip of the vector could fall within some "box" on your page. An example should help make this clear.

Suppose you are drawing a vector whose magnitude is assumed to be 150 units and direction is 0 degrees from the positive x-axis. Further, suppose the uncertainties in these measurements are 5 units for the magnitude and 2 degrees for the direction. You would draw this vector as shown below.
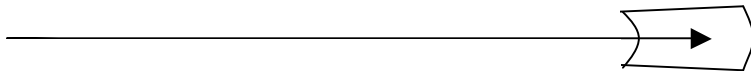
The center arrow represents the measured vector (150 units at 0 degrees). The "box" surrounding the tip of the vector has the following properties:

      1. Its straight side length is 10 units (145 to 155).

      2. The angle subtended by the box is 4 degrees (2 degrees above the vector and 2 degrees below it).

The box gives the area where the true vector may lie. When you draw vectors in this lab, you should use this method to determine the

We will defer an explanation of the algebraic method to later in the lab.

THREE VECTORS - NON-SYMMETRIC CASE

7. Pin the ring to the center of the table. Place one pulley at 90 degrees. Pass a string and weight hanger over this pulley with a total load of 90 grams. Position a second pulley at 0 degrees and a load of 120 grams. Place a third pulley at 217 degrees with a load of 150 grams. This should put the three forces in equilibrium. Test the equilibrium via the method in step 3 to be certain. These two forces at 90 and 0 degrees now represent the component resolution of the 150 grams of a force at 37 degrees. (Refer to the section with two vectors.) Thus these two forces have the same effect as the force of 150 gram at 37 degrees.

8. Next, before you change the configuration, use a protractor and ruler to make a scale drawing of the force vector diagram representing equilibrium. Set the scale so that the diagram comes as close as possible to filling the page. Be sure to indicate the scale on your drawing. In most cases a scale of 1 cm = 20 g will be appropriate. Include the uncertainties determined in the error analysis section above.

Draw each vector at the proper angle with lengths proportional to the forces (or mass). Since the force is just the product of the mass and the gravitational acceleration g, we may use the value of the masses to represent the forces and scale the drawing in terms of grams. Draw the vectors with the tail of each beginning at the origin of your coordinate system (which should be near the center of your paper).

Graphical Addition of Three Vectors

Next add the force at 90 degrees to the force at 0 degrees by adding the two vectors together head to tail or by making the parallelogram and find the resultant. How does the resultant compare to the third force?

9. Draw a new vector diagram with all of the vectors aligned head to tail so that you will have a triangle that should close if the three forces are in equilibrium. Draw the vector at 0 degrees from the origin of your coordinate system. Using the tip of this vector as a new origin, draw the vector pointing at 90 degrees. Finally, using the tip of the second vector as a new origin, draw the third vector. Ideally, you should have a closed triangle with one corner on the true origin of your graph. In reality, your triangle may fail to close. If so, determine by what amount of force the triangle fails to close. If this force fits into an uncertainty box, then you can claim that the triangle closes within the sensitivity of the experiment.

Component Addition of Three Vectors

10. Using trigonometry, calculate the x- and y-components of each of your vectors. Record this information in a table. Bear in mind that all angles must be measured from the same reference line (generally the positive x-axis). The sum of the x-components should be zero, as should the sum of the y-components.

11. Repeat the procedure above with the 150 g mass moved to a position of 225 degrees. Adjust the masses at 0 and 90 degrees until you can achieve equilibrium. What did you find for the two masses?

12. Draw a vector diagram for the situation of step 11 in the manner that you drew the diagram in step 8. Also compute the components from trigonometry. The sum of x- and y-components should be zero separately (as in step 10). This time, however, you may not achieve this. We must determine how the uncertainty in your measurements propagates to this sum before we can say anything about the validity of the experiment.

You should have values for Δm and $\Delta\theta$. You want a value for the uncertainty in the sum of vector components. If we represent the x-component uncertainty as Δx and the y-component uncertainty as Δy, and we know some rules of calculus we can determine how Δx and Δy depend on Δm and $\Delta\theta$. We will state the result, without proof.

$$\Delta x = \sqrt{[(\Delta m*(\sin\Theta_1+\sin\Theta_2+\sin\Theta_3))^2+(\Delta\Theta*(m_1\cos\Theta_1+m_2\cos\Theta_2+m_3\cos\Theta_3))^2]}$$

The subscripts 1, 2, and 3 refer to the three vectors. The formula for Δy is similar with the roles of sine and cosine exchanged. One important point is $\Delta\theta$ must be expressed in radians for this formula to be valid. If your values for Δx and Δy exceed the absolute values of the sums of the x- and y-components, then you can claim the forces are in equilibrium. Is this the case for your experiment?

13. Now choose three different masses, hang them on the weight hangers and adjust all of the angular positions until you reach an equilibrium. In this case you should not have any two pulleys making a 90 degree angle. Now make vector diagrams for this new situation as you did in both steps 8 and 9.

14. Calculate the x-components and determine if they sum to zero within the uncertainty. Similarly, calculate the y-components and determine if they sum to zero within the uncertainty.

QUESTIONS

1. What does equilibrium mean? What does it mean to resolve a vector into components?

2. If the ring were stationary but not centered about the pin, would the forces be in equilibrium? Why or why not?

3. If the force table were moving with a constant velocity would the results of this lab be affected? Explain.

4. If the force table were accelerated horizontally would the results of this lab be affected? Explain.